Downlink Precoding for Multiuser Spatial Multiplexing MIMO System Using Linear Receiver

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Abstract—**In this paper, we propose a novel unitary downlink precoding design scheme for multiuser (MU) spatial multiplexing (SM) multiple-input multiple-output (MIMO) systems. With the perfect channel state information (p-CSI) available at the transmitter and the linear decoder at the receiver, we construct the cost function based on the minimum average probability of vector symbol error (APVSE) and give the design method of the precoding matrices. Our proposed precoding matrices can completely eliminate co-channel interference (CCI) for each user at the transmitter, and each terminal user will eventually observe an interference-free single-user (SU) channel, thus simplify the decoding of each terminal user. Finally, we derive the upper bound APVSEs of several schemes for performance comparison and the latter simulation results have shown that our proposed downlink precoding for MU SM MIMO system obtains the almost same performance as the SU precoding system.**

Keywords-multiuser; MIMO; precoding; spatial multiplexing

I. INTRODUCTION

Spatial multiplexing (SM) is a simple technique that allows multiple-input multiple-output (MIMO) wireless systems to obtain high spectral efficiencies by dividing the bit stream into multiple substreams. Because these substreams are independently modulated, spatial multiplexing is more easily implemented than comparable space-time trellis or block codes. Unfortunately, the lack of spatial redundancy makes spatial multiplexing susceptible to rank deficiencies in the MIMO channel matrix. Linear precoding, a technique that divides the data stream into M substreams where M is smaller than the number of transmit antennas and then spreads the vector over the transmit antennas by a matrix multiplication, can reduce the probability of error by combatting rank deficiency problems [1]-[3].

Previous work in precoding has concentrated on the single user (SU) case where the precoding scheme assumes perfect channel state information (p-CSI) at the transmitter or focuses on limited feedback techniques such as channel quantization or limited feedback signal design [4]-[7]. In [8], a novel unitary precoder is proposed for the downlink transmission of an Alamouti space-time block coded MU wireless system in which p-CSI is available to the base transceiver station. By the using of precoder at the transmitter, the co-channel interference (CCI) at each mobile user is effectively precancelled and thus enables simple SU space-time block decoding.

In this paper, we propose a novel unitary downlink precoding design scheme for MU SM MIMO System. With the p-CSI available at the transmitter and the linear decoder at the receiver, the proposed precoder which is designed as a function of the channel and the linear decoder employed is able to completely eliminate CCI for each user, and the interference cancellation is exclusively carried out at the transmitter where the complexity costs can be shared by all users, thus enables simple SU SM linear decoding. Also, from the point of view of the average probability of vector symbol error (APVSE), our MU precoding for SM MIMO system obtains the almost same performance as SU system. The latter simulation results validate the rightness of our theoretical analysis.

This paper is organized as follows. In Section II the MU SM system model is introduced. In Section III we propose the cost function for the precoding matrices and give the method of construction of these matrices. In Section IV, performance analysis of our proposed scheme is derived and the simulation results are shown to demonstrate the performance of our proposed precoder. Section V gives the conclusions. In this paper, bold typeface, lower case letters (e.g., $\boldsymbol{x}$) represent vectors, bold typeface, upper case letters (e.g., $\boldsymbol{X}$) represent the matrices. $\boldsymbol{I}_{\mathrm{m}}$ is an $m\times m$ identity matrix. Superscripts $(\bullet)^T$, $(\bullet)^*$, $(\bullet)^{-1}$, and $(\bullet)^+$ denote vector transpose, complex conjugate transpose, matrix inverse and matrix pseudo-inverse, respectively. $[\boldsymbol{A}]_{l,l}^{-1}$ denotes the $(l,l)^{\text{th}}$ entry of matrix $\boldsymbol{A}^{-1}$, $\lambda(\boldsymbol{A})$ denotes the singular value of $\boldsymbol{A}$.

II. SYSTEM MODEL

The MU SM system under consideration is described

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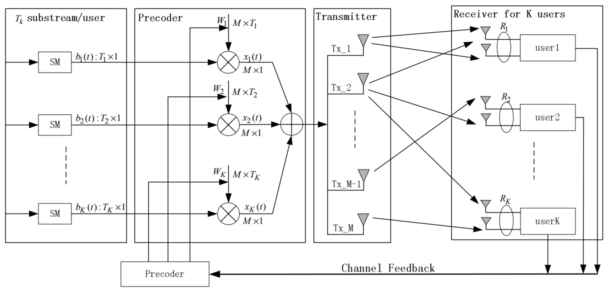

Fig. 1. Block diagram of the MU SM system with precoding: perfect feedback is assumed with $\{\boldsymbol{H}_k\}_{k=1}^{K}$ exactly known at the transmitter for precoder design.

by Fig. 1. The bit streams of each user is modulated independently using the same constellation Ω. This yields a symbol vector for the k^{th} user at time t $\boldsymbol{b}_k(t)=[b_{k,1}(t)\ \ b_{k,2}(t)\ \ \cdots\ \ b_{k,T_k}(t)]^T$, $k=1,2,\cdots,K$. For convenience we will assume that $\boldsymbol{E}[\boldsymbol{b}_k(t)\boldsymbol{b}_k^*(t)]=\boldsymbol{I}_{T_k}$.

For the k^{th} user, the symbol vector $\boldsymbol{b}_k(t)$ is then precoded by an $M\times T_k$ matrix $\boldsymbol{W}_k(t)$ yielding a length M vector $\boldsymbol{x}_k(t)=\sqrt{\frac{\varepsilon_k}{T_k}}\boldsymbol{W}_k(t)\boldsymbol{b}_k(t)$ where ε_k is the transmit energy of user k, M is the number of transmit antennas, and $M>T_k$. After precoding, the q^{th} transmit antenna transmits the q^{th} entry of $\boldsymbol{x}_k(t)$, $x_{k,q}(t)$.

The channel gain $h_{p,q}^{k}(t)$ for the k^{th} user between transmit antenna q and receive antenna p is modeled as random variable. We assume that the channel gain between each pair of transmit and receive antennas is distributed according to $CN(0,1)$, and if $m\neq p$ or $n\neq q$ then $h_{m,n}^{k}(t)$ is independent of $h_{p,q}^{k}(t)$.

The signal is received at one of R_k receive antennas of user k. We will assume throughout the paper that $R_k\geq T_k$. The signal received at the p^{th} receive antenna is added with a complex Gaussian noise $n_{k,p}(t)$ distributed as $CN(0,\sigma_n^2)$ where $n_{k,m}(t)$ is independent of $n_{k,p}(t)$ for $p\neq m$.

For the user k, this formulation allows the baseband, discrete time equivalent signal seen at the receiver to be written as

$$\boldsymbol{r}_k(t)=\boldsymbol{H}_k(t)\sum_{i=1}^{K}\boldsymbol{x}_i(t)+\boldsymbol{n}_k(t). \tag{1}$$

In (1), $\boldsymbol{H}_k(t)$ is a $R_k\times M$ matrix with $h_{p,q}^{k}(t)$ at entry (p,q) and $\boldsymbol{n}_k(t)=[n_{k,1}\ (t)\ \ n_{k,2}(t)\ \ \cdots\ \ n_{k,R_k}(t)]^T$.

For convenience we omit the time index t. In this paper, the received symbol vector $\boldsymbol{r}_k$ can be easily decoded with a linear decoder. A linear decoder can obtain the modified vector $\boldsymbol{y}_k=\boldsymbol{G}_k\boldsymbol{r}_k$ where $\boldsymbol{G}_k$ is a $T_k\times R_k$ matrix.

In this system, we assume that channel matrices $\{\boldsymbol{H}_k\}_{k=1}^{K}$ are perfectly available at the transmitter, either through reverse channel estimation in time-division-duplex (TDD) or feedback in frequency-division-duplex (FDD). We also assume that $\boldsymbol{H}_k$ associated with the k^{th} user is also perfectly known at the corresponding receiver, but they are not required to be known at other users. At the transmitter, the precoding matrix $\boldsymbol{W}_k$ is designed to precancelled CCI relative to the k^{th} user using some sort of performance criterion. Next, we will derive the cost function for the design of precoding matrices according to the minimum APVSE and give the design method of these precoding matrices.

III. DOWNLINK PRECODING

A. Construction of Cost Function for Precoding Matrices

In order to construct the cost function to obtain the precoding matrices $\boldsymbol{W}_k$ of the k^{th} user, from (1), assume that we have obtained the CCI-free received signals of the user k as follows

$$\boldsymbol{r}_k=\boldsymbol{H}_k\boldsymbol{x}_k+\boldsymbol{n}_k=\sqrt{\frac{\varepsilon_k}{T_k}}\boldsymbol{H}_k\boldsymbol{W}_k\boldsymbol{b}_k+\boldsymbol{n}_k. \tag{2}$$

For the linear receiver, we will characterize the APVSE performance using the substream with the minimum signal to noise ratio (SNR) following the results given in [2]. For the linear zero-forcing (ZF) decoder, we have

$$\boldsymbol{G}_k\boldsymbol{r}_k=\sqrt{\frac{\varepsilon_k}{T_k}}\boldsymbol{G}_k\boldsymbol{H}_k\boldsymbol{W}_k\boldsymbol{b}_k+\boldsymbol{G}_k\boldsymbol{n}_k. \tag{3}$$

In (3), $\boldsymbol{G}_k=(\boldsymbol{H}_k\boldsymbol{W}_k)^+$. So, from [2], the SNR of the l^{th}, $l\in(1\cdots T_k)$ substream is given by

$$SNR_l^{(ZF)}=\frac{\varepsilon_k}{T_k\sigma_n^2[\boldsymbol{W}_k^*\boldsymbol{H}_k^*\boldsymbol{H}_k\boldsymbol{W}_k]_{l,l}^{-1}}. \tag{4}$$

For the ZF receiver,

$$\left\{\begin{aligned}\max_{1\leq l\leq T_k}[\boldsymbol{W}_k^*\boldsymbol{H}_k^*\boldsymbol{H}_k\boldsymbol{W}_k]_{l,l}^{-1}&=\max_{1\leq l\leq T_k}\boldsymbol{e}_l^*[\boldsymbol{W}_k^*\boldsymbol{H}_k^*\boldsymbol{H}_k\boldsymbol{W}_k]^{-1}\boldsymbol{e}_l\\&\leq\max_{z:\|z\|_2=1}\boldsymbol{z}^*[\boldsymbol{W}_k^*\boldsymbol{H}_k^*\boldsymbol{H}_k\boldsymbol{W}_k]^{-1}\boldsymbol{z}\\&=\lambda_{\max}([\boldsymbol{W}_k^*\boldsymbol{H}_k^*\boldsymbol{H}_k\boldsymbol{W}_k]^{-1})\\&=\lambda_{\min}^{-2}(\boldsymbol{H}_k\boldsymbol{W}_k).\end{aligned}\right. \tag{5}$$

In (5), e_l is the l^{th} column of I_{T_k}. These results can be substituted into (4) to obtain

$$\begin{cases} SNR_{\min}^{(ZF)} = \min_{1\le l\le T_k} SNR_l^{(ZF)} \\ \qquad \ge \lambda_{\min}^2(\boldsymbol{H}_k\boldsymbol{W}_k)\dfrac{\varepsilon_k}{T_k\sigma_n^2}. \end{cases} \tag{6}$$

So, from [2], we can now bound the APVSE P_k as

$$\begin{cases} P_k = 1-\prod_{l=1}^{T_k}(1-P_l) \\ \quad \le 1-(1-P_{l_{\min}})^{T_k} \\ \quad \approx T_k P_{l_{\min}} \\ \quad \le T_k N_e Q(\sqrt{SNR_{\min}^{(ZF)}\dfrac{d_{\min}^2}{2}}). \end{cases} \tag{7}$$

In (7), $d_{\min}$ is the transmit minimum distance and N_e is the number of nearest neighbors of Ω [2]. So, from (6)-(7), we will get P_k smaller, we must make $\lambda_{\min}(\boldsymbol{H}_k\boldsymbol{W}_k)$ larger for the given Ω and ε_k/σ_n^2.

Problem Statement: Given fixed transmit power for each user, the objective is to design $\{\boldsymbol{W}_k\}_{k=1}^K$ so that CCI is completely precancelled at transmitter, meanwhile ensuring minimum APVSE. This problem aims to find $\{\boldsymbol{W}_{k,opt}\}_{k=1}^K$ such that

$$\boldsymbol{W}_{k,opt} = \arg\max_{\boldsymbol{W}_k\in C^{M\times T_k}} \lambda_{\min}(\boldsymbol{H}_k\boldsymbol{W}_k) \tag{8}$$

subject to

$$\begin{cases} \boldsymbol{W}_k^*\boldsymbol{W}_k = \boldsymbol{I}_{T_k} & k=1,2,\cdots,K \\ \boldsymbol{H}_i\boldsymbol{W}_k = 0 & i,k=1,2,\cdots,K, i\ne k. \end{cases} \tag{9}$$

Constraint-1 in (9) ensures a constant transmission power for the k^{th} user, and constraint-2 ensures that users cause no interference to each other. From (8), the optimum precoding matrix $\boldsymbol{W}_k$ is the key parameter to determine the APVSE.

B. *Design Method of Precoding Matrices*

To simplify the notation, denote the congregate interfering channel transfer matrix of user k as $\overline{\boldsymbol{H}}_k = (\boldsymbol{H}_1^*\cdots\boldsymbol{H}_{k-1}^*\boldsymbol{H}_{k+1}^*\cdots\boldsymbol{H}_K^*)^*$, so, the constraint-2 in (9) is then simplified to finding $\{\boldsymbol{W}_k\}_{k=1}^K$ such that $\overline{\boldsymbol{H}}_k\boldsymbol{W}_k = 0$, $k=1,2,\cdots,K$, which has a number of solutions. In this paper, we write $\boldsymbol{W}_k$ as $\boldsymbol{W}_k = (\boldsymbol{I}_M - \overline{\boldsymbol{H}}_k^+\overline{\boldsymbol{H}}_k)\boldsymbol{D}_k$, $\boldsymbol{D}_k$ is an $M\times T_k$ unitary matrix. It is easy to verify that $\overline{\boldsymbol{H}}_k\boldsymbol{W}_k$ is equal to zero. To satisfy transmit power constraint-1, Gram-Schmidt orthogonalization (GSO) can be performed with respect to the column vectors of $\boldsymbol{I}_M - \overline{\boldsymbol{H}}_k^+\overline{\boldsymbol{H}}_k$, in our simulations, we apply QR decomposition to implement GSO of $\boldsymbol{I}_M - \overline{\boldsymbol{H}}_k^+\overline{\boldsymbol{H}}_k$. Now, the performance optimization problem in (8) can be transformed to select $\{\boldsymbol{D}_k\}_{k=1}^K$ to maximize $\lambda_{\min}(\boldsymbol{H}_k\boldsymbol{W}_k)$, i.e.,

$$\boldsymbol{D}_{k,opt} = \arg\max_{\boldsymbol{D}_k\in C^{M\times T_k}} \lambda_{\min}(\boldsymbol{H}_k(\boldsymbol{I}_M - \overline{\boldsymbol{H}}_k^+\overline{\boldsymbol{H}}_k)\boldsymbol{D}_k). \tag{10}$$

At the transmitter, from [7], for the known and quasi-static channel realization $\boldsymbol{G}_k = \boldsymbol{H}_k(\boldsymbol{I}_M - \overline{\boldsymbol{H}}_k^+\overline{\boldsymbol{H}}_k)$, the $\boldsymbol{D}_{k,opt}$ can be obtained through singular value decomposition (SVD) of matrix $\boldsymbol{G}_k$. Now, we give the design of precoding matrix $\boldsymbol{W}_k$ for the k^{th} user as follows

$$\begin{cases} \overline{\boldsymbol{H}}_k = (\boldsymbol{H}_1^*\cdots\boldsymbol{H}_{k-1}^*\boldsymbol{H}_{k+1}^*\cdots\boldsymbol{H}_K^*)^* \\ \text{QR decomposition of } \boldsymbol{I}_M - \overline{\boldsymbol{H}}_k^+\overline{\boldsymbol{H}}_k :\to \boldsymbol{Q} \\ \textit{SVD of } \boldsymbol{G}_k = \boldsymbol{H}_k\boldsymbol{Q} = \boldsymbol{V}_L\boldsymbol{\Sigma}\boldsymbol{V}_R^* :\to \boldsymbol{V}_R \\ \boldsymbol{D}_k = \overline{\boldsymbol{V}}_R : \textit{the first } T_k \textit{ columns of } \boldsymbol{V}_R \\ \boldsymbol{W}_{k,opt} = \boldsymbol{Q}\boldsymbol{D}_k. \end{cases} \tag{11}$$

IV. PERFORMANCE ANALYSIS AND SIMULATION RESULTS

A. *Performance Analysis*

From section III, the upper bound APVSE of user k is expressed as

$$P_k \le T_k N_e Q(\sqrt{\lambda_{\min}^2(\boldsymbol{H}_k\boldsymbol{W}_k)\frac{\varepsilon_k}{T_k\sigma_n^2}\frac{d_{\min}^2}{2}}). \tag{12}$$

For performance comparison, we also give the upper bound APVSEs of SU non-precoding 2×2 (2 transmit and 2 receive antennas) system and SU precoding 4×2 system with 2 substream using linear ZF receiver.

For the SU non-precoding 2×2 system, the upper bound APVSE is given as

$$P_{SU-Non\,\Pr e} \le 2N_e Q(\sqrt{\lambda_{\min}^2(\boldsymbol{H}_1)\frac{\varepsilon_1}{2\sigma_n^2}\frac{d_{\min}^2}{2}}). \tag{13}$$

For the SU precoding 4×2 system, the upper bound APVSE is given as

$$P_{SU-\Pr e} \le 2N_e Q\left(\sqrt{\lambda_{\min}^2(\boldsymbol{H}_1\boldsymbol{W}_1)\frac{\varepsilon_1}{2\sigma_n^2}\frac{d_{\min}^2}{2}}\right). \qquad (14)$$

In (14), $\boldsymbol{H}_1$ is the 2×4 channel matrix with $h_{p,q}^1(t)$ at entry (p,q). We apply QR decomposition to $\boldsymbol{H}_1 = \boldsymbol{V}_L\boldsymbol{\Sigma}\boldsymbol{V}_R^*$ and obtain $\boldsymbol{W}_1 = \overline{\boldsymbol{V}}_R$, $\overline{\boldsymbol{V}}_R$ is the first two columns of $\boldsymbol{V}_R$.

In (12)-(14), the SNR at per receive antenna of per user is equal to $10\lg(\varepsilon_k/\sigma_n^2)$, ε_k denotes the total transmit energy of user k of MU system and ε_1 denotes the total transmit energy of SU system. For fair comparison, the SNRs of these schemes must be set to be same.

B. Setting of Simulation Parameters and Results

In the section, $d_{\min} = \sqrt{2}$ for QPSK and $d_{\min} = 2/\sqrt{10}$ for 16QAM. The N_e can be neglected because the APVSEs are dominated by the Q function. For performance comparison, Monte Carlo is implemented to obtain the various curves.

In Fig. 2 we plot the APVSEs of the four schemes at a spectral efficiency 4b/s/Hz using QPSK. For the SU 2×2 system, the ZF receiver does not obtain any diversity gain. For the SU 4×2 system, the ZF receiver obtains notable performance gain due to the use of precoding at the transmitter. Relative to the SU 4×2 system, the MU 4×2 with 2 substream system obtains the almost same performance due to the good design of our precoding matrices at the transmitter, thus precancelling the CCI. In Fig. 3 we repeat the above experiments at a spectral efficiency 8b/s/Hz using 16QAM. From the simulation curves, we achieve the same conclusions as above.

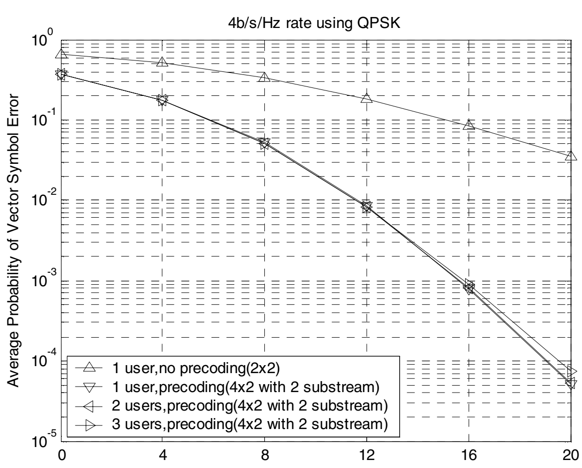

Fig. 2. APVSE comparison of SU and MU using QPSK.

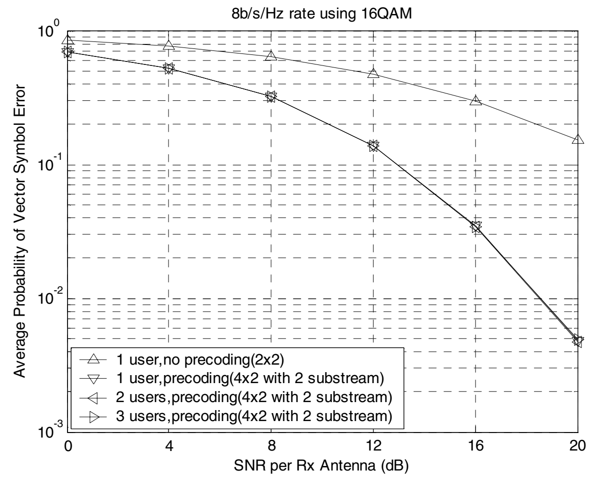

Fig. 3. APVSE comparison of SU and MU using 16QAM.

V. CONCLUSIONS

This paper proposed a novel unitary downlink precoding design scheme for MU SM MIMO system. With the channel knowledge available at the transmitter and the ZF decoder at the receiver, our proposed precoding matrices completely precancelled CCI between mobile users at the transmitter and thus simplified the decoding of each terminal user. For performance comparison, we derived the upper bound APVSEs of several schemes and the latter simulation results showed that our proposed downlink precoding for MU SM MIMO system obtains the almost same performance as the SU precoding system.

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